9th International Conference on Music Perception and Cognition

Alma Mater Studiorum University of Bologna, August 22-26 2006

Some considerations on algorithmic music and madrigals of Gesualdo da Venosa

Biancamaria Criscuolo
High School "L.Valla"
Castellammare di Stabia (NA), Italy
e-mail: bia1@virgilio.it

Francesco Russo
Mathematics Department
University "Federico II", Naples, Italy
e-mail: francesco.russo@dma.unina.it

ABSTRACT.
The object of this work is an algorithmic study of the madrigals of Carlo Gesualdo da Venosa. Here we construct a simulation model which allows to compose and classify an arbitrary madrigal of the Renassaince period. Gesualdo summarized the most important harmonic techniques to write a Renassaince madrigal, so we have translated these harmonic criteria in a suitable mathematical language, using linear systems. On the other hand a similar approach can be extended to madrigals of other times, giving different numerical values to the coefficients of the associated linear system.

In: M. Baroni, A. R. Addessi, R. Caterina, M. Costa (2006) Proceedings of the 9th International Conference on Music Perception & Cognition (ICMPC9), Bologna/Italy, August 22-26 2006.©2006 The Society for Music Perception & Cognition (SMPC) and European Society for the Cognitive Sciences of Music (ESCOM). Copyright of the content of an individual paper is held by the primary (first-named) author of that paper. All rights reserved. No paper from this proceedings may be reproduced or transmitted in any form or by any means, electronic or mechanical, including photocopying, recording, or by any information retrieval systems, without permission in writing from the paper's primary author. No other part of this proceedings may be reproduced or transmitted in any form or by any means, electronic or mechanical, including photocopying, recording, or by any information retrieval system, without permission in writing from SMPC and ESCOM.

INTRODUCTION

The present work is subdivided in four sections and it describes an algebraic-geometric simulation model about the Madrigals of Carlo Gesualdo da Venosa. This model can be extended to Mottets and Madrigals of an arbitrary musical period, since they are harmonically characterized by a special technique so called *imitatio in Cantus Firmus*.

Section 1 gives preparatory notions about Harmony and Contrapunctus. Here we see how is defined a Madrigal, summarizing some well-known results, which can be found with details in the references.

Introducing time-frequency diagrams in Section 2, we are able to formalize in a mathematical way the Cantus Firmus of a Madrigal. Section 3 is completely devoted to a mathematical approach of the Madrigal. We get to Equations **(3.1)**, **(3.2)**, **(3.3)**, **(3.4)** which completely classify a Madrigal, thanks to the specific affinity of the euclidean space, which is associated to a Madrigal.

During the process of writing of the Equations **(3.1)**, **(3.2)**, **(3.3)**, **(3.4)** we will need of an algorithm which is described in Russo (2004).

1. CARLO GESUALDO AND HIS MADRIGALS.

Almost all polyphonic musical forms such as *Canons*, *Fugues* and *Chorals* are characteristic not only by the *instrumental style*, but also by the *vocal style*. Many times a polyphonic musical form arises from as a vocal form which is successively adapted in an instrumental form, thanks to various reasons (see Einstein (1949), Jardine & others (1996), Nielsen (1980), Reese (1954)). Under this point of view it is clear why a same contrapunctistic tecnique is common to certain instrumental and vocal forms: typical examples are the works of the Fiamming School during the XV and XVI

Another classical problem, in the passage from vocal style to instrumental style, regards how to council the necessities of the text with the harmony of the piece. A good rule is that the text determines the fixionomy of the composition without modifying its structural nature, that is, its contrapunctus. In particular the *vocal Fugues* solve this problem with a repetition of the text, which has a metric near to the theme of the Fugue. In the vocal Canons the question Text-Music is easier, because the imitation involves contemporary the text and the music.
Mottet and Madrigal are special Renassaince vocal forms: they have not a corresponding pure instrumental version because Text-Music link is here very hard to clarify. A definition of Mottet is the following: a vocal form constructed on a liturgic text which has not *Cantus Firmus* (principal motive) in choral books. Similar harmonic definitions can be found in Nielsen (1980), Karolyi (1969).
Roughly speaking the most important aim of a Mottet consists to elaborate successive entires of voices and respective verses, to obtain an edifying atmosphere for the listener. A deep analysis on this musical form is documented by Einstein (1949), Gray & others (1926), Jardine & others (1996), Karolyi (1969), Nielsen (1980), Reese (1954). When the verse does not contain religious theme, we obtain the Madrigal, so an harmonic definition of the Madrigal could be given like that musical composition written with the same contrapunctistic laws of the Mottet but whose text has Cantus Firmus which does not regard liturgic matters.
Renassaince and Baroque Madrigals have many harmonic difficulties, they often test new instruments and techniques of composition. Under a poetic point of view, they show a special taste for the description. There are Madrigals which speak about nature, rain, sun, but others treat theme as love, death and similar state of mind. During the Baroque period in Italy it was born a famous trend called *Teoria degli affetti*, which interprets the spirit of this poetry. For a detailed studies on the matter one can see Einstein (1949), Gray & others (1926), Jardine & others (1996).
Carlo Gesualdo was certainly a famous author of Renassaince Madrigals and his tecniques summarized all the knowledge about this musical form. His books of Madrigals treat many affects and they contributed to define the famous querelle of C.Monteverdi (see Einstein (1949), Gray & others (1926), Jardine & others (1996)) on the importance of the Text-Music link. The introduction at the VI book of Monteverdi as soon as the introduction at the I book of Gesualdo help to understand the problem which the harmonists had to compose music and text, during the XVI century, but it is also an explaining about the taste of those years.

2.TIME –FREQUENCY DIAGRAMS.

An intuitive way to visualize a translation of the usual euclidean plane is offered by the following figure (see also Choquet (1964), Weyl (1952)).
It is interesting to note what happens, composing with translations in a suitable way. It is possible to discover a logic of this kind in many musical pieces and also in Madrigals, when we formalize certain musical circumstances in mathematical language.

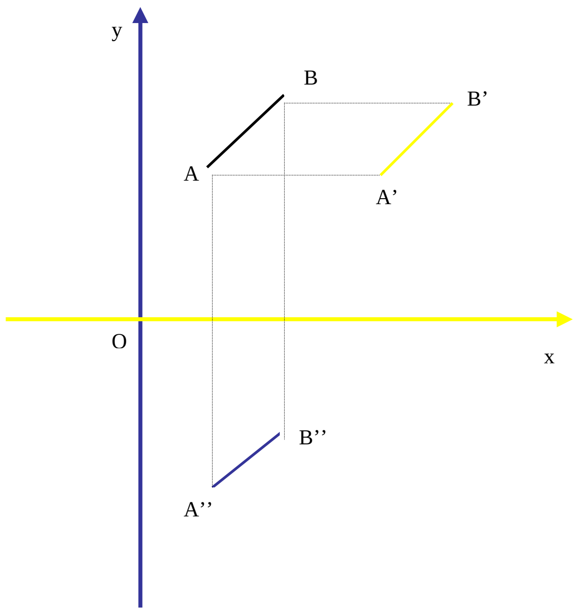

Figure 2.1. *Translation.* A’B’ is translated by AB long the x axis , in the creasing direction. A’’B’’ is translated by AB long the y axis, in the decreasing direction.

Introducing a monometric orthogonal reference frame in the euclidean plane, we can visualize the temporal evolution of an hand which plays the ascending scale of C major in the central octave of the piano. Fixed a unity of time, for istance the quaver, we grade the x axis (*Duration Axis* or *duration*) with integer multiple of the fixed unity. On the y axis (*High Axis* or *height*) we grade according to the frequencies of the white tastes of the central octave of the piano. Roughly speaking, each sound is characterized by hight, intensity and timbre, in particular the height is an acoustic size which is measured in Hertz (Benson (200), Frova(1990), Mazzola (2002), Russo (2004)). The range which competes to the white tastes of the central octave of the piano goes from 264 to 520 Hertz in the sequence do=C=264Hz, re=D=297Hz, mi=E=330Hz, fa=F=352Hz, sol=G=396Hz, la=A=440Hz, si=B=495Hz, do=C=520Hz.
We remark that the black tastes and other frequencies between two consecutive white tastes of the central octave of the piano are not showed only to semplify the discourse.
Fig. 2.2 describes a common exercise of piano: the ascending scale of C major, played by a single hand in the central octave. Another common exercise, the scale of C major for thirds, is described by Fig. 2.3.

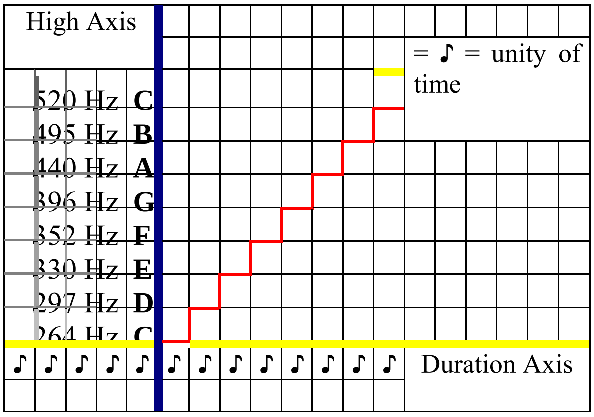

Figure 2.2. *Graphic representation in a time-frequency diagram of the ascending scale of C major*. In this monometric orthogonal reference frame is used the quaver as unity of time for the Duration Axis (duration), while on the High Axis (height) there is a band of frequencies associated with the white tastes of the central octave of the piano.

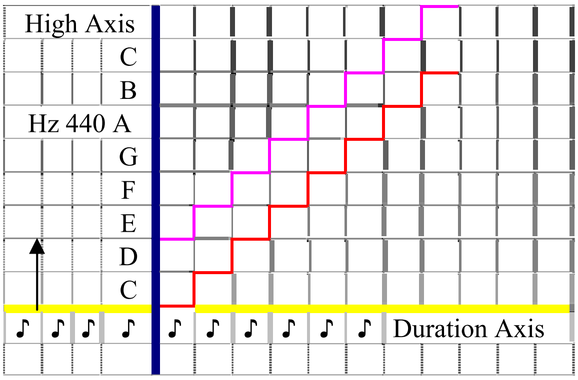

Figure 2.3.*Graphic representation in a time-frequency diagram of the ascending scale of C major by thirds in parallel motus.*

= left hand
= right hand

Starting at the same instant, the right hand repeats the motive of the left hand in a more acute register (distance of superior third) along High Axis. In mathematical language (see also Fig.1.1) the melody played by right hand (2nd-voice) is translated by the melody played by left hand (1st-voice) along High Axis. The natural question is to see what happens when 2 or more instruments (voices) are playing, describing translations in their melody. This is an intuitive idea about a *Canon* and Galante (1998), Mazzola (2002), Russo (2004), Scimemi (1983, 1997) make a more detailed study under this mathematical point of view. More generally we are speaking about the idea of an harmonic tecnique so called *imitation*; it allows to treat a Gesualdo's Madrigal with linear systems and algorithms.

3. ANALYSYS OF GESUALDO'S MADRIGAL N.1 VOL.III.

Initial part of a Madrigal (*ingressum* or *incipit*) is subjected to declaration of the Cantus Firmus by means the voices which are singing. Successively they go on without a fixed rule and after this moment there is neither a contrapunctistic restriction nor a periodicity about the composition, which give a structural scheme. In particular Gesualdo madrigals present many harmonic exceptions, which are typical about Gesualdo's style, but also here it is possible to find this feature in incipit. There is an exposition of the Cantus Firmus in the voice which firstly starts to sing and after few moments the other voices repeat it. Imitation is not rigorous as soon as in the Canon (*severa imitatio*), so, before to Cantus Firmus entire, there is a small transient ad libitum or some intervals are not all faithfully respected. According to Italian style of madrigals, these transients are personal and they characterize the author i.e. Gesualdo makes a great use of chromatisms.

If we restrict the attention only about Cantus Firmus entire, it is possible a treatment with linear systems like in Russo (2004), whose solution is given by elementary algorithms (see Assyag&others (2002), Benson (2003), Mazzola (2004),). Moreover the choice of the Cantus Firmus is submitted by poetic rules and one can predict the qualitative andament of the melody in terms of time-frequency diagrams, once that it is given the affect which is described by the madrigal.

The following list is referred to a description of the most common Affects in the Renassaince and Baroque period.

1. Dead and Melancony.
   There are aggravament and dilatation.
   The line of the Cantus Firmus is described by a function *f(x)* with *f'(x)<0*, being *x* the value of duration and *f'(x)* the derived function of *f* in the point *x*.
   The range of *x* is determined by the interval of durations, related with Cantus Firmus.

2. Enjoy, Nature, Happyness, Edificant matters.
   There are diminution and great use of chromatisms.
   The line of the Cantus Firmus is harmonically complicated and it can be described by a function *f(x)* with *f'(x)>0*, being *x* the value of duration and *f'(x)* the derived function of *f* in the point *x*.

The range of x is determined by the interval of durations, related with Cantus Firmus.

3. Other Affects.
   There are both aggravaments and diminutions, which depend by the style of the author.
   The line of the Cantus Firmus is described by a function $f(x)$ with $f'(x) \leq 0$ or $f'(x) \geq 0$, being x the value of duration and $f'(x)$ the derived function of f in the point x.
   The range of x is determined by the interval of durations, related with Cantus Firmus.
   If the Affect is near to previous point 1, we often note that $f'(x) \leq 0$, if the Affect is near to previous point 2, we often note that $f'(x) \geq 0$.

A detailed list of Affects in Renassaince period is remanded to Einstein (1949), Giamatti (1984), Gray & others (1926), Hale (1994), Jardine & others (1996), Plumb (1961), Reese (1954), Watkins (1991).
A precise definitions about the melodic function f we refer to Mazzola (2002).
At this point it is possible to analyze a Madrigal of Carlo Gesualdo under a mathematical point of view; for simplicity we choose Madrigal n.1 of vol.III.
It is useful to recall that a *phrase* of a madrigal is harmonically defined by the melody which is song between two moments of silence. It is common practice to consider the start and the end of a piece as moments of silence (for details Karolyi (1969), Nielsen (1980)).

Text of the Madrigal n.1 vol.III of Carlo Gesualdo:
Voi volete ch'io mora,
Nè mi togliete ancora,
questa misera vita.
E non mi date incontr'a morte a ita.

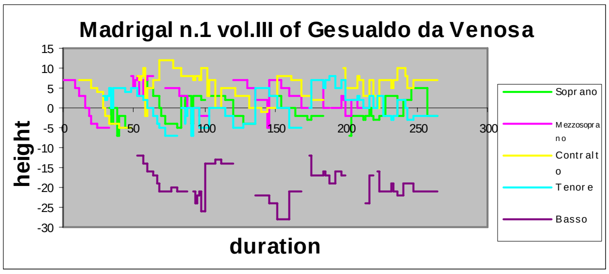

Figure 3.1. *Graphic representation in a time-frequency diagram of the Madrigal n.1 vol.III of Carlo Gesualdo:*

1) The unity of time for the duration is the least value of time which is present in the score, that is, the semiquaver. Each other note has duration which is multiple of semiquaver.
2) On the height we have adopted a frequency's band which covers the extension of 2 octaves on the piano between 150 and 1046 Hz. This band describes black and white tastes (semipitches of equable temperament).
3) Origin of the reference frame is A at 440 Hz.

Fixed Mezzosoprano as 1st-voice, the imitation between the Cantus Firmus of Mezzosoprano and Cantus Firmus of Contralto is rigorous and it is governed by the following equation, which describes a translation of Π :

$$\tau_1 \quad \begin{cases} d_2 = d_1 + 12 \\ h_2 = h_1 \end{cases} \qquad \textbf{(3.1)}$$

where the unknown d_2 and h_2 show the duration and the height of the Cantus Firmus of Contralto. The values d_1 and h_1 are fixed and they show the duration and the height of the Cantus Firmus of Mezzosoprano. Equation **(3.1)** means a simple fact which we listen: Contralto starts after 12 semiquaver pauses at the same height of Mezzosoprano. The translation vector of τ_1 is (12, 0), which is parallel with Duration Axis in the reference frame which we have previously described.
The imitation between the Cantus Firmus of Mezzosoprano and the Cantus Firmus of Soprano is rigorous and it is governed by the following equation

$$\tau_2 \quad \begin{cases} d_3 = d_1 + 56 \\ h_3 = h_1 \end{cases} \qquad \textbf{(3.2)}$$

where the unknown d_3 and h_3 show the duration and the height of the Cantus Firmus of Soprano and the values d_1 and h_1 are fixed and they show the duration and the height of the Cantus Firmus of Mezzosoprano. Also here we are saying that in a time-frequency diagram, Soprano starts after 56 semiquaver pauses at the same height of Mezzosoprano.
The imitation between the Cantus Firmus of Mezzosoprano and the Cantus Firmus of Tenor is not rigorous and it is governed by the following equation

$$\tau_3 \quad \begin{cases} d_4 = d_1 + 48 \\ h_4 = h_1 + 5 \end{cases} \qquad \textbf{(3.3)}$$

where the unknown d_4 and h_4 show the duration and height of the Cantus Firmus of Tenor and the values d_1 and h_1 are fixed and they show the duration and height of the Cantus Firmus of Mezzosoprano. Here Tenor starts after 48 semiquaver pauses in a more acute range of height with respect to Mezzosoprano. This difference is measured by 5 semipitches in the introduced reference frame.
The imitation between the Cantus Firmus of Mezzosoprano and Cantus Firmus of Basso is governed by the following equation

$$\tau_4 \quad \begin{cases} d_5 = d_1 + 52 \\ h_5 = h_1 - 12 \end{cases} \qquad \textbf{(3.4)}$$

where the unknown d_4 and h_4 show the duration and the height of the Cantus Firmus of Basso. The values d_1 and h_1 are fixed and they show the duration and the height of the Cantus Firmus of Mezzosoprano. Here Basso starts after 52 semiquaver pauses in a more deep range of height with respect to Mezzosoprano. This difference is measured by 12 semipitches in the introduced reference frame.

Classification of Madrigal n.1 vol.III of Carlo Gesualdo.
According with the construction of a melodically affine Canon in Russo (2004), Equation **(3.1)** was born, choosing the first three non collinear points in Fig. 3.1 on Mezzosoprano i.e (0,7), (8, 5), (12, 3). We impose at **(3.1)** to send these points in the first three non collinear points on Contralto i.e. (12,7) , (20, 5), (24, 4). Then we obtain a linear system of 6 equations in 6 unknowns which has the unique solution given by the values $\alpha_2 = 1$, $\beta_2 = 0$, $\gamma_2 = 0$, $\delta_2 = 1$, $d_{0,2} = 16$, $h_{0,2} = 0$ (see Russo (2004)). In this way we obtain **(3.1)**. A similar argument can be applied to Equations **(3.2), (3.3), (3.4)**.
(3.1) gives a translation τ_1 which is parallel with Duration Axis in the time-frequency diagram previously described. Structural Theorem for isometries of Π (see Choquet (1964) or Weyl (1954)) allows to say that τ_1 can be reduced at the product of two successive reflections which have parallel axis whose direction is ortoghonal with Duration Axis. This description is characteristic for τ_1 .
A similar argument can be applied to the translations τ_2 , τ_3 , τ_4 .
We are able to classify Madrigal n.1 vol.III of Carlo Gesualdo thanks to its Cantus Frimus which is governed by the 4-pl of translations of Π (τ_1 , τ_2 , τ_3 , τ_4).

4. CONCLUSIONS.

The method described to analize Madrigal n.1 vol.III of Carlo Gesualdo can be extended to all madrigals with an arbitrary number of voices (n-voices canon, n positive integer) where it is adopted an imitation tecnique on Cantus Firmus. As before we fix the 1^{st}-voice, then we write (n-1)-equations of the kind **(3.1)** in their general form:

$$\begin{cases} d_2 = a\ d_1 + b \\ h_2 = c\ h_1 + d \end{cases} \qquad \textbf{(4.1)}$$

being a , b, c, d real numbers and where the unknown d_2 and h_2 show the duration and hight of the 2^{nd} -voice. The values d_1 and h_1 are fixed and they show the duration and height of the 1^{st}-voice. The recognition of isometries and affinities in Equation **(4.1)**, thanks to Structural Theorem on the isometries of Π , means to classify harmonically the Madrigal.
On the other hand it is interesting to note that a similar approach to Madrigals allows a method to compose. The assignment of the coefficients a , b, c, d and the values d_1 and h_1 in **(4.1)** lead to construct the Cantus Firmus of a Madrigal with two voices, resolving the associated linear system. It is clear what is the argument when we want to write the government equation for the Cantus Firmus of a Madrigal with n-voices .

A treatment with linear system which we have described can be simulated by elementary algorithms (Gauss Algorithm).

ACKNOWLEDGMENTS

The second author is indebted with Proff. P.Fergola and G.Guerriero of the Mathematics Department of Naples for their suggestions.

REFERENCES

Arons, A.B. (1992). *Guida all'insegnamento della Fisica*, Bologna: Zanichelli.

Assayag, G., Feichtinger, H.G. & Rodrigues, J.F. (2002). *Mathematics and Music.* Heidelberg: Springer.

Benson, D. (2003). *Mathematics and Music*. Cambridge: Cambridge University Press.

Choquet, G.(1964). *L'insegnamento della Geometria*. Milano: Feltrinelli.

Einstein, A. (1949). *The Italian Madrigal.* Princeton: Princeton University Press.

Fauvel, J., Flood, R. & Wilson, R. (2003). *Music and Mathematics*. Oxford: Oxford Univ. Press.

Fergola, P. (2002). Atti del Convegno. In: Fergola, P. (Ed.), *Proceedings of Musica, Infanzia e Adolescenza Dicembre 2002.* Napoli, Italy: RISMA.

Frova,A.(1990). *Fisica nella Musica*. Bologna: Zanichelli.

Furinghetti,F.(1990). Matematica oggi, dalle idee alla scuola. Genova: Mondadori.

Galante,D. (1998). Aspetti didattici dello studio delle trasformazioni geometriche. *Seminari G.R.I.M. 1998*.

Giamatti, A.B. (1984). *Exile and Change in Renaissance Literature*. New Haven-London: Yale University Press.

Gray C. & Heseltine P. (1926). *Carlo Gesualdo, Musician and Murderer.* London.

Hale,J.(1994). *The Civilization of Europe in the Renaissance.* New York: Touchstone.

Jardine, L. & Goods,W. (1996). *A New History of the Renaissance*. London: Macmillan.

Karolyi, O. (1969). *La grammatica della musica*. Torino: Einaudi .

Mazzola,G.(2002). *The Topos of Music*. Heidelberg: Birkhauser.

Nielsen, R. (1980). *Le forme musicali*. Bologna: Bongiovanni.

J. H. Plumb, *The Horizon Book of the Renaissance* (1961). USA: American Publishing Corporation.

Robinson,D. (1980). *A Course in the Theory of Groups*. New York: Springer.

Reese,G. (1954). *Music in the Renaissance*. New York: W.W. Norton & Co.

Russo,F. (2004). *La musica algoritmica e l'Offerta Musicale di J.S.Bach*. Grottaminarda: Delta 3.

Scimemi,B. (1983). Musica e Aritmetica e Buon Temperamento. *Archimede, n. 1-2*, pp. 3-20.

Scimemi,B.(1997). Contrappunto musicale e trasformazioni geometriche. In: Emmer,M. (Ed.), *Lettera Matema-tica Pristem*, (Atti del convegno su Matematica e Cultura). Venezia, Italy.

Tannery,P. (1902). Du rôle de la musique greque dans le développement de la mathématique pure. *Mémoires scientifiques, vol. 3.*

Watkins,G.(1991). *Gesualdo: The Man and His Music.* Oxford: Oxford University Press.

Weyl, H. (1952). *Symmetry*. Princeton: Princeton University Press.